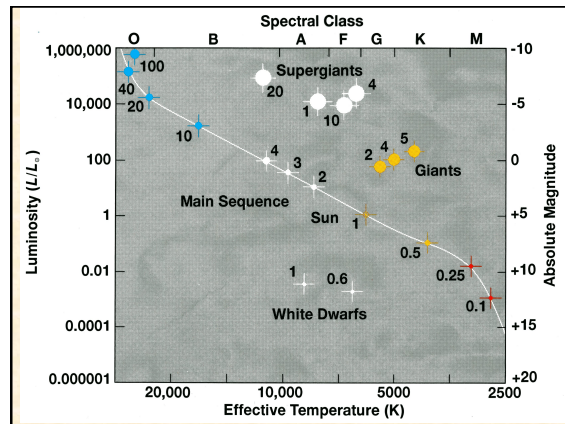


- LAST TIME: Using Spectral Line Positions
 - HR Diagram
 - Binary Stars
 - Mass Determination
- TODAY: Mass-Luminosity Relation
 - mass measurements from binary stars
 - mass-luminosity relation
 - T, L, M, R
- FRIDAY: EXAM #1 !!!



Measuring Mass Using Binary Stars

- Kepler's Third Law: $1/M_T \sim a^3/P^2$
- But we are interested in individual masses of each component, not total mass
- P is easy to measure (but for visual binaries, it can take 10's or 100's of years)
- What about a? (demo: hula hoop)

Visual Binary

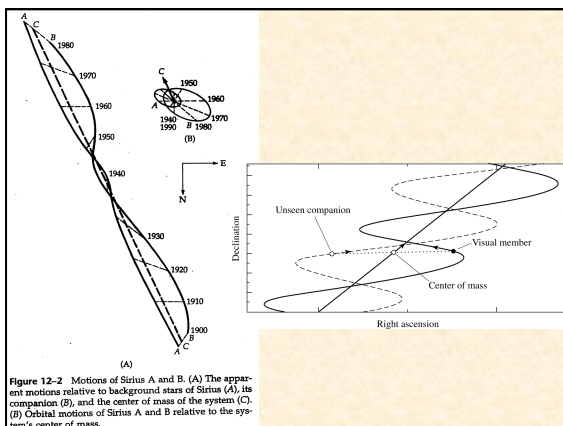
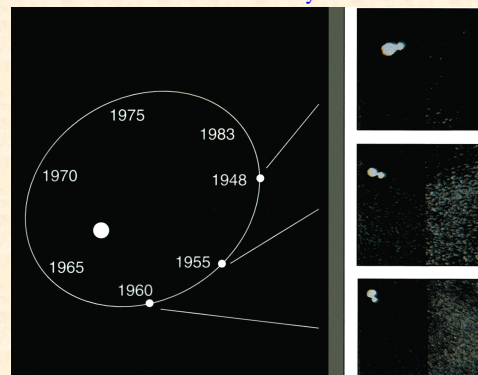


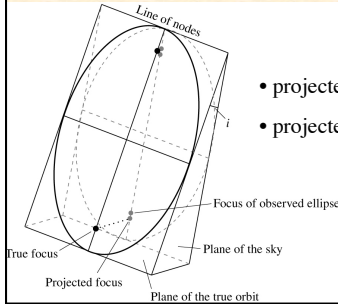
Figure 12-2 Motions of Sirius A and B. (A) The apparent motions relative to background stars of Sirius (A), its companion (B), and the center of mass of the system (C). (B) Orbital motions of Sirius A and B relative to the system's center of mass.

1) Visual Binaries

- Mass ratio given by relative semi-major axes:

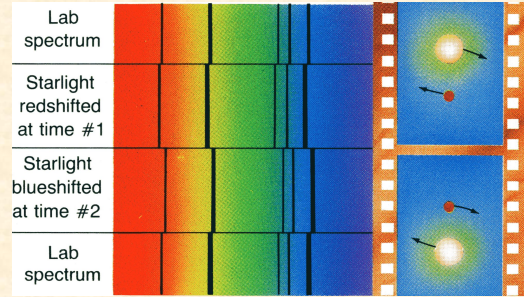
$$m_1/m_2 = \alpha_1/\alpha_2; \quad \text{where } \alpha = a/d$$
- distance not required if you can see both stars and the center of mass (just use angular separation)
- if orbit in plane of sky and distance is known, can set up 1-body problem with reduced mass $\mu = m_1 m_2 / (m_1 + m_2)$ orbiting immobile total mass
 - $a = a_1 + a_2$
 - Kepler's 3rd law gives $m_1 + m_2$
 - combine mass ratio and mass sum ->
 - solve for m_1 & m_2 individually (2 eq., 2 unknowns)

- if not in plane of sky, you can still fiddle with more careful observations (accelerations, line of nodes, longitude of ascending node, etc) to solve for orbit and therefore mass



- projected orbit still an ellipse
- projected foci not true foci

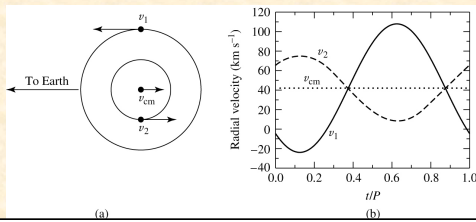
Spectroscopic Binary



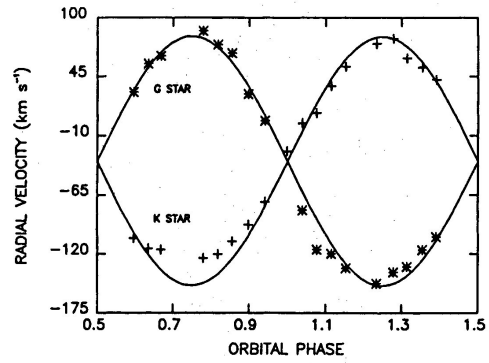
single-lined v. double-lined spectroscopic binaries

2) Spectroscopic Binaries

- Special Case: circularized; eclipsing binaries
 - eclipsing binary: $\sin i = 1$ ($i=90$) circular: $e=0$
 - $m_1/m_2 = v_2/v_1$
 - in general v is not constant and is projected v_r
 - $v=2\pi a/P$; a =radius of circular orbit
 - Kepler's 3rd $\rightarrow m_1+m_2$

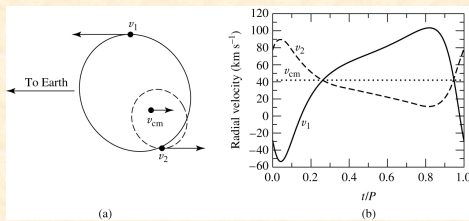


What can you conclude about this system?

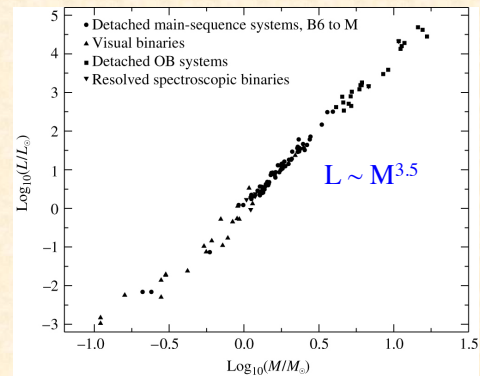


- More general case, total mass given by:
 - $m_1+m_2 = (P/2 \pi G) (rv_1 + rv_2)^3 / \sin^3 i$
 - if we see both stars and know i , we can solve
 - if we see both stars but not i , only get $w/\sin^3 i$
 - if we only see 1 star only get "mass function":

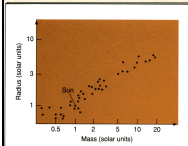
$$m_2^3 / (m_1+m_2)^2 \sin^3 i = (P/2 \pi G) rv_1^3$$



Plot Mass v. Other Parameters...



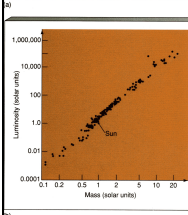
Mass-Luminosity Relation



$$L \sim M^{3.5}$$

- L comes from conversion of mass into energy
- so Lifetime \sim Mass/Luminosity

$$\text{Max. Lifetime} \sim M^{-2.5}$$



- Mass range .1 \rightarrow 20 M_{\odot}
 - $M \sim 20$ 25 million years
 - $M \sim 1$ 10 billion years
 - $M \sim 0.1$ 1 trillion years

Properties We Have “Measured”

- from apparent brightness, position, “color”
 - distance
 - proper motion
 - luminosity
 - effective temperature
 - radius
- from Doppler shifts of spectral lines
 - radial velocity
 - space velocity
 - mass

These measured properties cover a large range:

	min	max	range
Luminosity	10^{-4}	10^4	10^8
Radius	10^{-2}	10^3	10^5
Mass	10^{-1}	10^2	10^3
Temperature	2500	25000	10^1

(L, R, M in Solar Units; T in Kelvin)