Lec #19: 5 OCT 2011

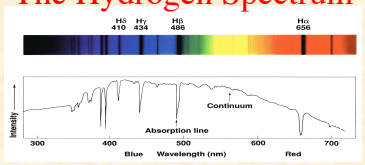
TODAY: Line Positions and Strengths

- Balmer Formula
- Bohr Atom
- Hydrogenic Ions
- Transition Rates and Line Strengths

NEXT: Atomic Physics. II.

- Modifications required for >1 electron
- Atomic Structure, Selection Rules, Atomic "Terms"

The Hydrogen Spectrum



- Balmer fit pattern with
  $1/\lambda = R_H (1/4 - 1/n^2)$, n=3,4,5,...
- Led to discovery of more lines. Generalize to
  $1/\lambda = R_H (1/ m^2 - 1/n^2)$, m<n; both integers
  - m=1 Lyman series (uv)
  - m=2 Balmer series (visible)
  - m=3 Paschen series (ir)

The Bohr Atom (Hydrogen)

- Analog w/ gravity
  - $1/r^2$ central force
  - angular momentum = constant
  - but not just any constant; orbits "quantitized"
  - so we don't get Kepler's Laws
  - still satisfy "Virial Theorum" U=-2K
  - $L = \mu v r = nh/2\pi = n(h\text{-bar})$; n=positive integer
  - 2-body --> 1-body problem (reduced mass, μ)
  - circular orbits (centripital force = electric force)

- $F=q_1q_2/r^2=e^2/r^2$ (cgs)
- $\mu=m_em_p/(m_e+m_p) = 0.9994556\ m_e$
  ($m_p=1836m_e$)
- circular orbits: $\mu v^2/r = e^2/r^2$
- $KE = 1/2\ \mu v^2$ $\quad PE = -e^2/r$
- $E = KE+PE = -1/2\ e^2/r$
- use $L=\mu v r=nh/2\pi$ in place of μv to get...

- $r_n=\{h_b2/\ \mu\ e^2\}\ n^2 = a_0\ n^2$; $a_0 = 0.529$ Å
- so $r = a_0\ \ 4a_0\ \ 9a_0$ ...

- so $r = a_0\ 4a_0\ \ 9a_0$ ...

- Plug this into Energy equation to get...
- $E_n = -1/2\ e^2/r_n = -13.6\ eV\ 1/n^2$
- $E_n = -13.6, -3.4, -1.5, -0.9, ...$ eV

- $E_\gamma = E_U - E_L = 10.2, 1.9, 0.6, ...$ eV

- Turn this into wavelength with $E=hc/\lambda$ ...
- $1/\lambda = R_H \{1/n_L^2 - 1/n_U^2\}$

  where $R_H = \mu e^4/4\pi\ h_b^3 c = 109{,}677.5\ cm^{-1}$

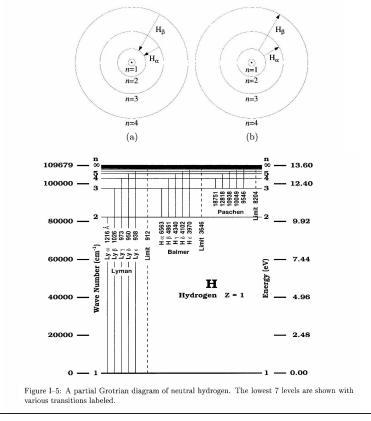


Figure 1-5: A partial Grotrian diagram of neutral hydrogen. The lowest 7 levels are shown with various transitions labeled.

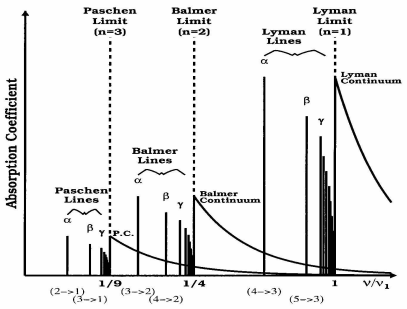


Figure I–3: Schematic of the b-f and b-b absorption coefficients of hydrogen.

- $1/\lambda = R_H \{1/n_L^2 - 1/n_U^2\}$

  where $R_H = \mu e^4/4\pi h_b^3 c$
- Generalize to any "Hydrogen-like" atom...
- $1/\lambda = R Z^2 \{1/n_L^2 - 1/n_U^2\}$

Hydrogen	109,677.5 cm^{-1}
Deuterium	109,707
Helium (He II)	109,722
Carbon (C VI)	109,733
Nitrogen (N VII)	109,733.5
Oxygen (O VIII)	109,734
Infinity	109,737.31

at H-alpha:

μ (not m_e) -> 2.5 Å

Deuterium shifted ~1.5 to blue

DeBroglie Wavelength

- Wave-Particle Duality isn't just for photons any more!
- Recall that photons have *Energy* E=hν=hc/λ and *Momentum* p= hν/c= h/λ
- They also carry *Angular Momentum* h/2π (h_b)
- Suppose λ_D=h/p for everything!
- For an n=1 electron, λ_D= 3.3 Å = 2π a_0

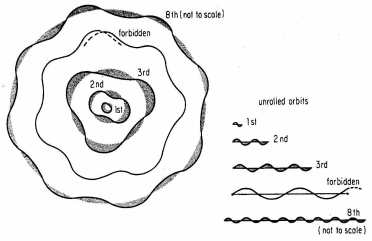


TABLE 6-2
Electron Wavelengths in Hydrogen

Orbit	Electron mass	Electron velocity	Electron wavelength
1	9.1×10^{-31} kg	2.18×10^6 m/sec	3.33 Å
2	"	1.09	6.66 Å
3	"	0.73	9.99 Å
4	"	0.54	13.32 Å
5	"	0.43	16.65 Å

Standing Waves

not accelerating electrons! (so no steady-state emission)

explains why orbits are quantitized

Important QM Fundamentals

- Classical physics breaks down on the size scale of atoms, but the following are still conserved
  - mass-energy
  - momentum
  - angular momentum
  - charge
- Heisenberg Uncertainty Principle
  - h-bar serves as a quantum of "action" (E*t or p*x)
  - $\Delta p \Delta x \sim h_b$ and $\Delta E \Delta t \sim h_b$
  - for an atom $\Delta x \sim \lambda_D$ --> electron "clouds"

What Can We Tell From Line Position?

- What elements are present (but relative strengths don't mean much, and absence of evidence is not evidence of absence).
- Doppler Shifts: systematic motion in the line of sight
- Doublets and Triplets: spin-orbit coupling; isotopic shifts; magnetic field
- We'll discover some more things later. But for now...

What Can We Tell From Line Strength?