Lec #22: 12 OCT 11 Spectral Line Profiles 4.3

Last Time: What can we Determine from Line Strength?

Today: What Can We Determine From Line Profiles?

- lifetime of level
- physical conditions (e.g. local gravity)
- systematic and random motions
- number of absorbers

Next: Radiative Transfer (Chapter 3)

Bound-Bound Rate Coefficients

(see 4.3.5 in LeBlanc)

- Excitation
  - B_{12} (photoexcitation)
  - C_{12} (collisional excitation)
- De-Excitation
  - A_{21} (spontaneous emission) s^{-1}
  - B_{21} (stimulated emission)
  - C_{21} (collisional de-excitation)

- *B* depends on mean intensity of radiation field, J_ν
- *C* depends on density of colliders (mostly n_e) and kinetic temperature

Level Populations: e.g. 2-level atom

- dn/dt = transitions IN - transitions OUT

  $dn_2/dt = n_1[B_{12}J_\nu(T_r)+n_eC_{12}(T_K)] - n_2[A_{21}+B_{21}J_\nu(T_r)+n_eC_{21}(T_K)]$

  $dn_1/dt = n_2[A_{21}+B_{21}(T_r)J+n_eC_{21}(T_K)] - n_1[B_{12}J(T_r) +n_eC_{12}(T_K)]$
- In 2-level atom

  n_1+n_2=constant and $dn_2/dt = - dn_1/dt$
- if dn/dt=0 (must be if only 2 levels) you can rearrange either of the above equations to get

  $n_2/n_1 = [B_{12}J_\nu +n_eC_{12}] / [A_{21}+B_{21}J_\nu +n_eC_{21}]$
  - depends on radiation field and electron density
  - and on rate coefficients

Luminosity in a Line (e.g Lα)

- L (erg/s) = transitions/sec * Energy of transition
  - $\Delta E = (E_2-E_1) = h\nu = hc/\lambda$
  - transitions per second = N_2/τ_2
    - where τ_2 = lifetime of excited state
    - and N_2 = # of electrons in excited state
- $N_2/V = n_2$ $N_H/V = n_H$
  - $N_2 = N_H(n_2/n_1) = n_HV (n_2/n_1)$
  - so it reduces to finding (n_2/n_1)
  - in thermal equilibrium, this is easy (and so are a lot of other things...)

- In *Thermal Equilibrium*
  - $J_\nu = B_\nu$ (Planck function)
  - Boltzmann distribution of level populations
  - every process strictly balance by its inverse ("Principal of Detailed Balance")
- "Einstein relations"

  $B_{21} = g_1/g_2\ B_{12}$

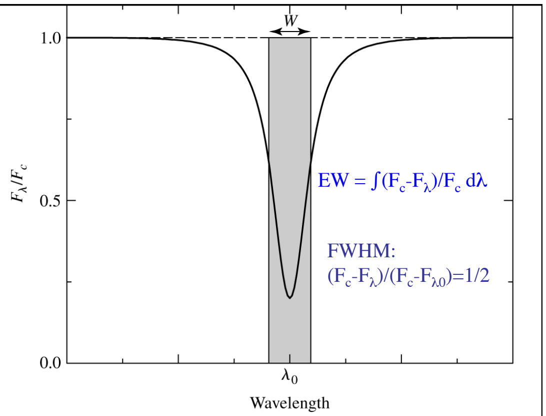
  $A_{21} = 2h\nu^3/c^2\ B_{12}$

  $C_{21} = g_1/g_2 \exp(\Delta E_{12}/kT)\ C_{12}$
  - These depend only on atomic parameters, so they should be valid for any temperature!
  - g_1 and g_2 are "statistical weights" (# of degenerate electron states in each level; e.g. $2n^2$)

How Do We Measure Line Strength?

- Experimentally, we don't know absolute intensity of "continuum"
- so we "normalize" the continuum
- But absorption is a fractional process anyway
- Use fractional depth, but line is not at 1 λ
- Must integrate depth over dλ (across line)
- "Equivalent Width" W (in Å) defined by (4.4.1)

  $W = \int (F_c-F_\lambda)/F_c\, d\lambda$
- width of rectangle with area equal to integrated area in line and height equal to continuum (1)



Determining the # of Absorbers, N_a

- need to use BOTH line strength (EW) and profile
  - input T, n
  - assume TE, use Boltzmann and Saha eq's to get n_j/n_i
  - if not TE, all is not lost ("Statistical Equilibrium")
  - atomic physics, lab measurements to get A_{ji} and relative probability of A_{ji} to all the other possible i's (e.g. $P_{32} \neq P_{31}$)
  - "oscillator strength", gf value
- Theoretical "Curve of Growth" for each species, transition, n, T, etc. combination (4.4.3)

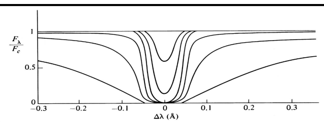


Figure 9.20 Voigt profiles of the K line of Ca II. The shallowest line is produced by $N_a = 3.4 \times 10^{11}$ ions cm^{-2}, and the ions are ten times more abundant for each successively broader line. (Adapted from Novotny, *Introduction to Stellar Atmospheres and Interiors*, Oxford University Press, New York, 1973.)

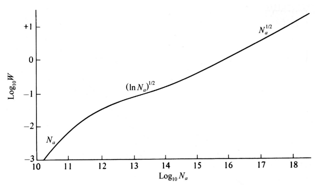


Figure 9.21 The curve of growth for the K line of Ca II. As N_a increases, the functional dependence of the equivalent width (W) changes. At various positions along the curve of growth, W is proportional to the functional forms indicated. (Data from Aller, *The Atmospheres of the Sun and Stars*, Ronald Press, New York, 1963.)

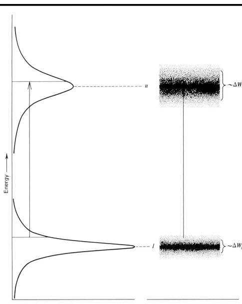
Linear part: $W \sim N_a$ (optically thin)

Flat part: $W \sim (\ln N_a)^{1/2}$ (saturated core)

Root N part: $W \sim N_a^{1/2}$ (pressure broadened)

What Determines Line Shape?

- Natural broadening
- Collisions
- Doppler shifts
  - systematic motions
  - random motions
- Electric and Magnetic Fields
- Instrumental resolution



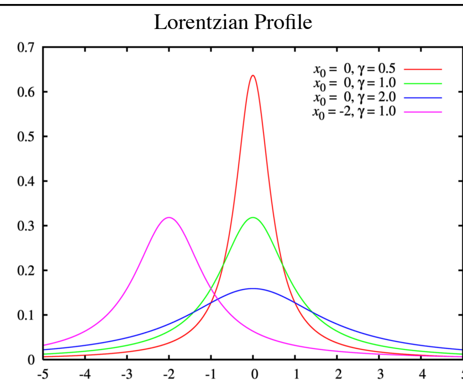
1. Natural Broadening

- $\Delta E \sim \hbar/\Delta t$
- $\Delta \nu = A_{21}/2\pi$

$$\phi^N(\nu)\, d\nu = \pi^{-1} \Delta^N / [(\nu-\nu_0)^2 + \Delta^{N2}]\, d\nu$$

$$\Delta^N = A_{21}$$

- "lorentzian profile", with very broad wings



2. Pressure (collisional) Broadening

- broadens levels; depends on...
  - lifetimes of levels and frequency of collisions
- has lorentzian shape and characteristic width

$$\phi^C(\nu)\, d\nu \sim \Delta^C/[(\nu-\nu_o)^2 + \Delta^{C2}]\, d\nu$$

$$\Delta^C \sim n\sigma\, (2kT/m)^{1/2}$$

- "Damping" from classical H.O. formation
  - radiative damping v. collisional damping
  - broad wings
  - n term "explains" *Luminosity Class* in MK scheme

3. Doppler Broadening

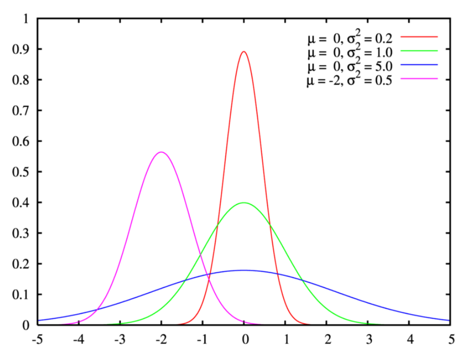
- Maxwellian distribution of absorber/emitter (atom) velocities
- Lorentzian in atom frame, but Doppler shift ($\Delta\lambda/\lambda = v_r/c$) -> gaussian profile

$$\phi^D(\nu)\, d\nu \sim 1/(\pi\Delta^D)\, e^{-[(\nu-\nu_o)/\Delta^D]^2}\, d\nu$$

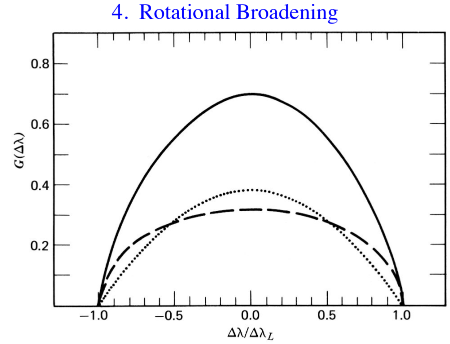
$$\Delta^D = (v_T/c)\, \nu_o = \nu_o/c\, (2kT/m)^{1/2}$$

  - m=mass of atom!
- What else can cause Doppler broadening?
  - turbulence
  - rotation, flows, pulsation (not necessarily gaussian)

Gaussian Profile



4. Rotational Broadening



Rotation profile looks more like a quadratic

Combined N+C+D Profile

- "Voigt" profile

$$\phi^V(\nu)\, d\nu \sim \pi^{-3/2}\, \{a/\Delta^D\} \int e^{-t^2}/[(\nu-\nu_o)/\Delta^D - t)^2 + a^2]\, dt$$

$$a = (\Delta^N + \Delta^C)/\Delta^D$$

  - a is Voigt parameter and t is "dummy variable"
  - a=>0 for gaussian a=>∞ for lorentzian
  - in astrophysics a ranges from 10^{-4} to 10^{-1}
- in terms of $x = (\nu-\nu_o)/\Delta^D$

$$\phi^D(x)\, dx \sim \pi^{-1/2}\, e^{-x^2}\, dx$$

$$\phi^V(x)\, dx \sim \pi^{-3/2}\, a \int e^{-t^2}/[(x-t)^2 - t^2) + a]\, dt$$