Lec #24: 19 Oct 2011 Chapters 3 & 4

This Week: Radiative Transfer. I.

- Specific Intensity, $I_\nu$
- Emission and Absorption coefficients
- Radiative Transfer Equation
- Optical depth
- Source function
- Opacity and Emissivity

Next Week: Stellar Atmosphere Models

## Combined N+C+D Profile

- "Voigt" profile

$$\phi^V(\nu)\, d\nu \sim \pi^{-3/2} \{a/\Delta^D\} \int e^{-t^2}/[(\nu-\nu_o)/\Delta^D - t)^2 + a^2]\, dt$$

$$a = (\Delta^N+\Delta^C)/\Delta^D$$

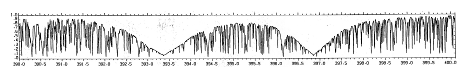
  - $a$ is Voigt parameter and t is "dummy variable"
  - $a \Rightarrow 0$ for gaussian $a \Rightarrow \infty$ for lorentzian
  - in astrophysics, $a$ ranges from $10^{-4}$ to $10^{-1}$
- in terms of $x=(\nu-\nu_o)/\Delta^D$

$$\phi^D(x)\, dx \sim \pi^{-1/2} e^{-x^2}\, dx$$

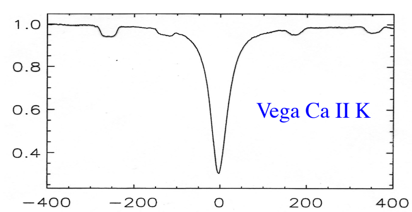
$$\phi^V(x)\, dx \sim \pi^{-3/2}\, a \int e^{-t^2}/[(x-t)^2 - t^2) + a]\, dt$$

## Deconvolution of Intrinsic Profile

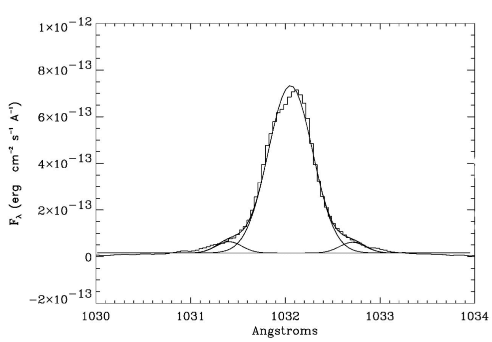
- That's not all, other things affect profile:
  - Zeeman broadening, hyperfine structure, "microturbulence"
  - instrumental resolution ("line spread function")
- How do these all combine to produce what we observe?
  - "CONVOLUTION" : multiply the Fourier Transforms of all the $\phi$'s (including instrumental) then do an inverse transform to get the combined $\phi(\nu)$
  - "DECONVOLUTION" : remove the effects of the $\phi$'s that we think we understand



Solar Ca II K & H

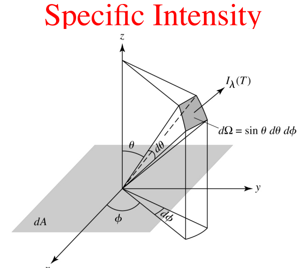



Vega Ca II K




Voigt Profile? Or Multiple Components?

## Specific Intensity




$dE_\nu = I_\nu\, dt\, dA\, d\Omega\, d\nu$

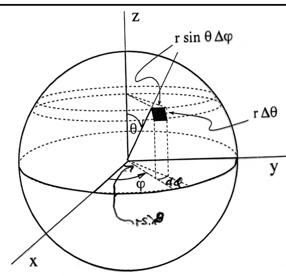
$I_\nu = dE_\nu / (dt\, dA\, d\Omega\, d\nu)$

*cgs units:* erg/s $cm^{-2}$ $sr^{-1}$ $Hz^{-1}$

*mks units:* W $m^{-2}$ $sr^{-1}$ $Hz^{-1}$

*in TE: $I_\nu = B_\nu$*

*(same as $B_\nu$)*




Solid angle $\Omega = \text{area}/r^2$

- e.g. complete sphere = $4\pi r^2 / r^2 = 4\pi$ "steradians" (sr)
- $dA = (r\, d\theta) * (r \sin\theta\, d\varphi) = r^2 \sin\theta\, d\theta\, d\varphi$
- $d\Omega = dA/r^2 = \sin\theta\, d\theta\, d\varphi$

## Mean Intensity

- $I_\nu$ is the "monchromatic" intensity
- $I_\nu$ is constant "along a ray" through vacuum
- Total intensity is $I = \int I_\nu\, d\nu$ over all frequencies
- "Mean Intensity"

$$J_\nu = (1/4\pi) \int I_\nu\, d\Omega$$

  - *average* over all angles, not total over all angles
  - same units as $I_\nu$
  - we saw $J_\nu$ in the rate equations for B-B transitions
  - $J_\nu = I_\nu$ if $I_\nu$ is isotropic ($B_\nu$ always isotropic)

## The Radiative Transfer Equation

- EMISSION
  - "volume emissivity" $j_\nu$ (erg/s cm$^{-3}$ sr$^{-1}$ Hz$^{-1}$)
    - $dE_\nu = j_\nu\, dV\, d\Omega\, dt\, d\nu$

$$dI_\nu/ds = j_\nu$$

(s is physical path through medium)

  - "emissivity per unit mass" $\varepsilon_\nu$ (erg/s g$^{-1}$ sr$^{-1}$ Hz$^{-1}$)
    - $dE_\nu = \rho\varepsilon_\nu\, dV\, d\Omega\, dt\, d\nu$

$$dI_\nu/ds = \rho\varepsilon_\nu$$

  - $j_\nu$ and $\varepsilon_\nu$ are not necessarily independent of $I_\nu$
    - but we can alter our definition to make it so...

- ABSORPTION
  - "absorption coefficient" $\alpha_\nu$ (cm$^{-1}$)

$$dI_\nu/ds = -\alpha_\nu I_\nu$$

  - "opacity" $\kappa_\nu$ (cm$^2$ g$^{-1}$)

$$dI_\nu/ds = -\rho\, \kappa_\nu I_\nu$$

  - cross sections... $\sigma_\nu$ (cm$^2$) & $n$ (cm$^{-3}$)

$$dI_\nu/ds = -\sigma_\nu\, n\, I_\nu$$

- SCATTERING
  - can either add or remove photons from beam, but they are still there in the gas (just a net change of direction)
  - might involve absorption and re-emission (in a different direction)
  - proportional to intensity, so if we include scattering in absorption coefficient, it can be negative

- Equation of Radiative Transfer

$$dI_\nu/ds = j_\nu - \alpha_\nu I_\nu$$

or...

$$dI_\nu/ds = -\rho(\varepsilon_\nu - \kappa_\nu I_\nu)$$

  - this is the simplest (and least useful) form
  - what does it mean?
  - "opacity" v. absorption coefficient

- Optical depth (dimensionless)

  $d\tau_\nu = \alpha_\nu \, ds$    $\tau_\nu = \int_o^s \alpha_\nu(s')ds'$

  pure absorption... $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$
  - physical, but not geometric
  - frequency dependent
- Source Function, $S_\nu = j_\nu / \alpha_\nu$
  - same units as $I_\nu$
  - often simpler and more physical to deal with net “source” rather than emission and absorption separately

## Source Function

- with $d\tau_\nu = \alpha_\nu \, ds$  and  $S_\nu = j_\nu / \alpha_\nu$
- RT Eq ($dI_\nu/ds = j_\nu - \alpha_\nu I_\nu$) can be written as:

$$dI_\nu/d\tau_\nu = - I_\nu + S_\nu$$

  - I approaches S
  - S --> $B_\nu$ (“thermal emission”)
  - determining S is equivalent to solving RT eq
  - but it can involve integrals, differentials
  - and lots of physics

- Some very simple solutions.
  1. No absorption ($\alpha_\nu$=0)

     $dI_\nu/ds = j_\nu$

     $I_\nu(s) = I_\nu(s_0) + \int_o^s j_\nu(s')ds'$

  2. No emission ($j_\nu$=0)

     $dI_\nu/ds = - \alpha_\nu I_\nu$

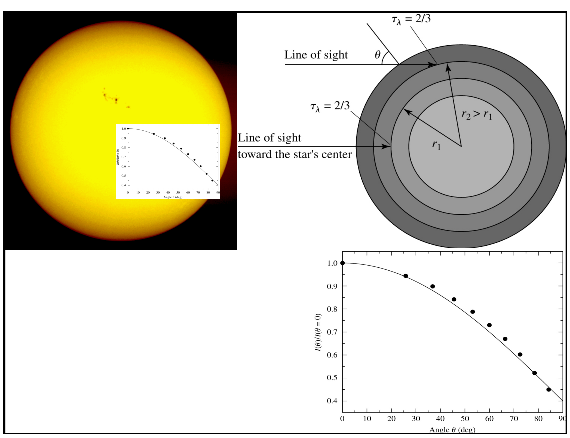
     $I_\nu(s) = I_\nu(s_0)\, e^{-\int \alpha_\nu(s')ds'}$   ($\int$ from 0 to s)

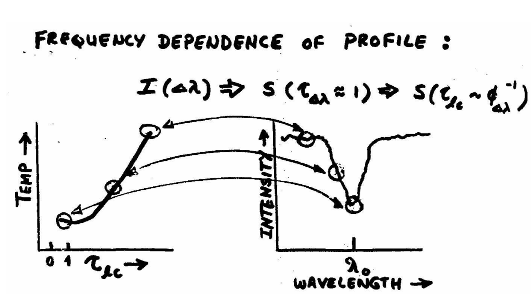
## Optical Depth

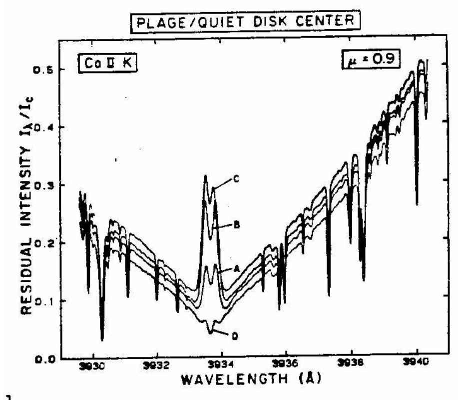
- $\tau_\nu$ (dimensionless)

  $d\tau_\nu = \alpha_\nu \, ds$    $\tau_\nu = \int_o^s \alpha_\nu(s')ds'$

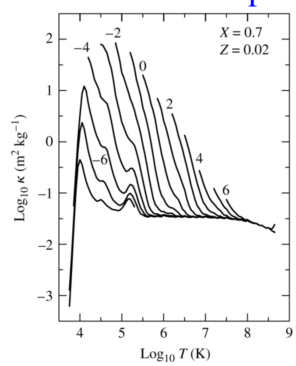
  pure absorption... $I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$
  - physical, but not geometric
  - frequency dependent
- What value of $\tau_\nu$ yields a 50:50 probability of escape?
  - $e^{-2/3} = 1/2$ ==> $\tau = 2/3$ plays a special role
- optically “thin”; optically “thick”
- Limb darkening (examples)
- Profile maps different heights, therefore temperatures, in an atmosphere (examples)






# Opacity




opacity coefficient: $\kappa_\nu \rho = \alpha_\nu$

- bound-bound
- bound-free
- free-free
- electron scattering

$H^-$ opacity: continuous below 16,400 Å; narrow temp. range; very important

"Rosseland Mean"