Lec #29: 31 October 2011 — Chapter 5

Part III. Stellar Structure & Evolution

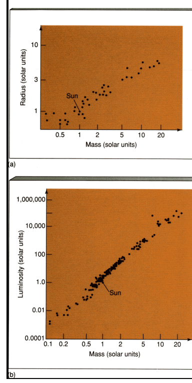
This Week: Modeling Stellar Structure (Chap 5)

- Equations of Stellar Structure
- Hydrostatic Equilibrium
- Energy *Transfer* Within Stars
- The Sun as a Star

Next: Energy Generation; Nucleosynthesis (Chap 6)

THEN: Stellar Evolution

Mass-Luminosity Relation

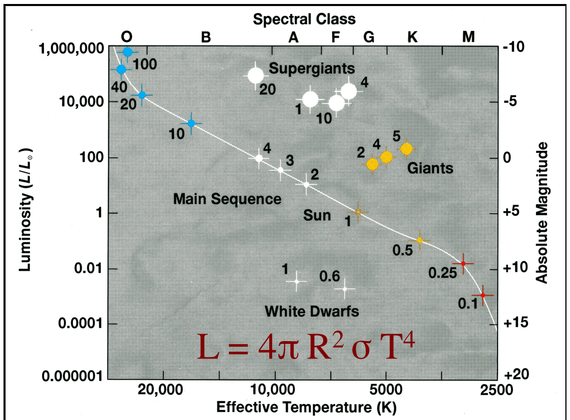


$L \sim M^{3.5}$

- L comes from conversion of mass into energy
- so Lifetime~Mass/Luminosity

Max. Lifetime ~ $M^{-2.5}$

- Mass range .1 -> 20 $M_\odot$
  - M~20 — 25 million years
  - M~1 — 10 billion years
  - M~0.1 — 1 trillion years



These measured properties cover a large range:

	min	max	range
Luminosity	10^{-4}	10^{4}	10^{8}
Radius	10^{-2}	10^{3}	10^{5}
Mass	10^{-1}	10^{2}	10^{3}
Temperature	2500	25000	10^{1}

(L, R, M in Solar Units; T in Kelvin)

$$L = 4\pi R^2 \sigma T^4$$

Equations of Stellar Structure

Strategy: simultaneously solve 4 or more *one-dimensional* differential equations to yield *global* parameters (L, M, T_{eff}, R)

1. **Pressure Balance** (Hydrostatic Equilibrium)
   - $dP/dr = - G M_r \rho(r) / r^2$
   - neglect flows; magnetic "pressure"
2. **Conservation of Mass** (Continuity Equation)
   - $dM_r/dr = 4\pi r^2 \rho(r)$
3. **Conservation of Energy**
   - $dL_r/dr = 4\pi r^2 \rho(r)\, \varepsilon(r)$
   - ε contains all net photon generation (~0 outside core)

4. **Energy Transport**
   - *Radiative*: $dT/dr = -3/4ac\ \kappa\rho(r)/T^3\ L_r/4\pi r^2$
   - *Convective*: $dT/dr = g(r)/c_P$ (adiabatic)
   - *Conductive*: not usually important for interiors ?!

- **Equation(s) of State**
  - P, κ, ε as functions of ρ, T, composition
  - e.g. *Ideal Gas* P = nkT
  - ideal gas doesn't always work
  - will deal with Fermi-Dirac and Bose-Einstein

- **Boundary Conditions**
  - r --> 0 — M_r and L_r --> 0
  - r --> ∞ — T, P, ρ --> 0
  - r --> R — r, T, M_r, L_r --> R, T_{eff}, M, L

- Result:
  - T(r) and ρ(r)
  - integrate ρ(r) to get mass
  - boundary conditions to get T_{eff} , Radius, etc.
  - integrate L_r to get total Luminosity
  - match observed global properties
- Next Step: (the Sun and main-sequence stars)
  - test with Solar Models and observations
  - see how these properties change with time
  - "Stellar Evolution" (explain HR diagram)
- Caveats: (real stars; more interesting stars)
  - winds, rotation, magnetic fields, binary stars
  - more complicated equations of state, better convection models, etc.