Lec #30: 2 November 2011 Chapter 5

This Week: Modeling Stellar Structure (Chap 5)

- ✓ Equations of Stellar Structure
- Hydrostatic Equilibrium
- Energy *Transfer* Within Stars
- The Sun as a Star

Next: Energy Generation; Nucleosynthesis (Chap 6)

THEN: Stellar Evolution

## Equations of Stellar Structure

Strategy: simultaneously solve 4 or more *one-dimensional* differential equations to yield *global* parameters (L, M, $T_{eff}$, R)

1. **Pressure Balance** (Hydrostatic Equilibrium)
   - $dP/dr = - G M_r \rho(r) / r^2$
   - neglect flows; magnetic "pressure" ; add $P_{rad}$
2. **Conservation of Mass** (Continuity Equation)
   - $dM_r/dr = 4\pi r^2 \rho(r)$
3. **Conservation of Energy**
   - $dL_r/dr = 4\pi r^2 \rho(r) \varepsilon(r)$
   - ε contains all net photon generation (~0 outside core)

4. **Energy Transport**
   - *Radiative*: $dT/dr = -3/4ac \ \kappa\rho(r)/T^3 \ L_r/4\pi r^2$
   - *Convective*: $dT/dr = g(r)/c_P$ (adiabatic)
   - *Conductive*: not usually important for interiors ?!

- **Equation(s) of State**
  - P, κ, ε as functions of ρ, T, composition
  - e.g. *Ideal Gas* P = nkT
  - ideal gas doesn't always work
  - will deal with Fermi-Dirac and Bose-Einstein

- **Boundary Conditions**
  - r --> 0 $M_r$ and $L_r$ --> 0
  - r --> ∞ T, P, ρ --> 0
  - r --> R r, T, $M_r$, $L_r$ --> R, $T_{eff}$, M, L

## The Balance of Gravity and Pressure

- Pressure Balance (e.g. Hydrostatic Equilibrium)

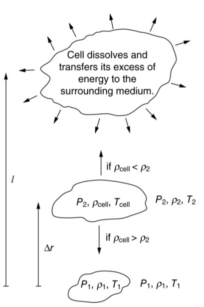
$$dP/dr = - G M_r \rho(r) / r^2$$

- Pressure "Gradient" must match gravity gradient to satisfy Hydrostatic Equilibrium
- To be complete, Pressure can also involve radiative pressure, magnetic field, "ficticious forces", etc.

$$dP/dr = - \rho(r) g(r) + 1/3aT^4$$

- g is defined *locally* ($GM_r/r^2$)
- $M_r$ is total mass contained within r

## Adiabatic Convection




- bubble expands without exchanging energy with environment; pressure balance
- if density inside bubble drops off faster with height than density in surrounding medium, bubble will rise
- $d \ln(P) / d \ln(T) > 2.5$ then stable against convection

## Equations of State

- need *Equation of State*
- e.g. "Ideal Gas"
  - PV=NkT P=(N/V)kT P=nkT
  - alternative form involving ρ=n * m
  - m is average mass of a gas particle
- "mean molecular weight" $\mu=m/m_H$
  - μ is temperature and composition dependent
  - $P_r = \rho_r k T_r / \mu_r m_H$
  - relates pressure with Temperature and composition at each radius

- Mass Fractions rather than numbers...

  $X=M_H/M$ $Y=M_{He}/M$ $Z=M_{metal}/M$

  $X+Y+Z=1$

  - e.g. neutral $1/\mu = X+(1/4)Y+<1/A>Z$
  - solar: X=.70, Y=.28, Z=.02, A~15.5 so $\mu$~1.3
  - e.g. 100% ionized $1/\mu = 2X+(3/4)Y+(1/2)Z$
  - solar: X=.70, Y=.28, Z=.02, A~15.5 so $\mu$~0.62
- Radiation Pressure $P_{rad}=(1/3)aT^4$
- Total Pressure = $P_{gas} + P_{rad}$ + ...

- Result:
  - T(r) and $\rho$(r)
  - integrate $\rho$(r) to get mass
  - boundary conditions to get $T_{eff}$ , Radius, etc.
  - integrate $L_r$ to get total Luminosity
  - match observed global properties
- Next Step: (the Sun and main-sequence stars)
  - test with Solar Models and observations
  - see how these properties change with time
  - "Stellar Evolution" (explain HR diagram)
- Caveats: (real stars; more interesting stars)
  - winds, rotation, magnetic fields, binary stars
  - more complicated equations of state, better convection models, etc.