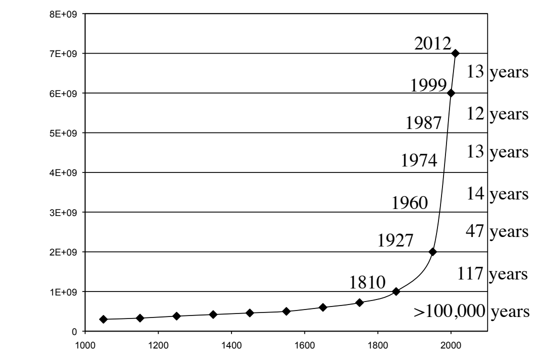
Lec #3: Energy Implications of Growth

<u>Previous:</u>
- Bartlett Video, Part 1: Mathematics of Growth
- Introduction to Course

<u>TODAY:</u>
- Discussion of Population Growth and its Implications for Resource Consumption
- Mathematics of Exponential Growth

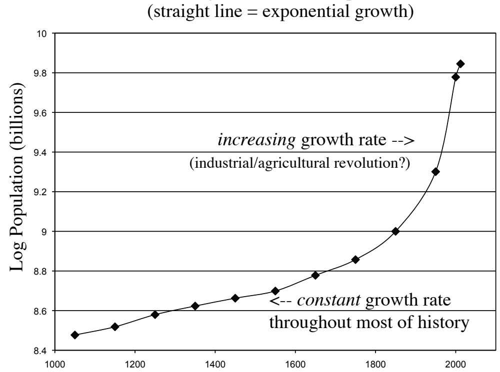
<u>NEXT WEEK: (finish reading Chapter 1)</u>
- Estimating the Remaining Lifetime of Fossil Fuels
- What causes an "energy crisis"?
- Can it be avoided?

Exponential Growth of World Population



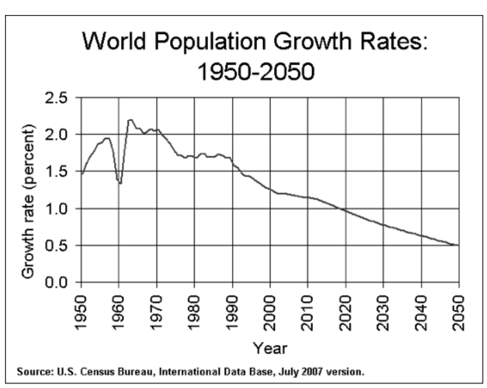
Caution: What do you "see" when you plot an exponential function?

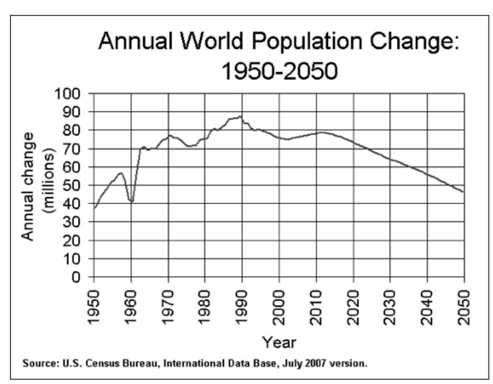
Semi-log Plot of World Population
(straight line = exponential growth)



Assuming constant growth rate of about 2% per year (doubling time = 35 years)

	Year	Total Number of people	Population Density (1/m²)
current	1998	$6x10^9$	$4x10^{-5}$
mass_people=mass_earth	3540	$7.5x10^{22}$	$1.5x10^8$
using 100% of solar energy	2600	$8.5x10^{14}$	1.6
using 100% incident on land w/ clouds	2500	$1.1x10^{14}$	0.2
using 10% through consumption	2345	$6.7x10^{11}$	$1.2x10^{-3}$
1/4 land arable; 50% food to animals	2140	$8.4x10^{10}$	$6.3x10^{-4}$
typical city			$6.2x10^{-4}$
Club of Rome - maximum		15-20 billion	$1.3x10^{-4}$
UN - maximum		11.5 billion	$8.6x10^{-5}$





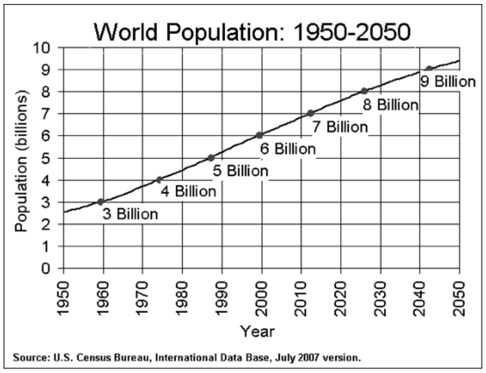


Table of Options

Increase Populations	Decrease Populations
Procreation	Abstention
Motherhood	Contraception/Abortion
Large Families	Small Families
Immigration	Stopping Immigration
Medicine	Disease
Public Health	
Sanitation	
Peace	War
Law & Order	Murder & Violence
Scientific Agriculture	Famine
Accident Prevention	Accidents
Clean Air	Pollution
Ignorance of the Problem	

What Causes Exponential Growth?

change proportional to current amount

- Example #1: Compound Interest
  - interest earned in 1 compounding period = fixed fraction of current amount (interest rate, I)
  - e.g. $1000 at 10%/year
  - $1000*.10=$100 total = $1100
    - = $\$1000*1.1 = \$1000*(1+.1)^1$
  - $1100*.10=$110 total = $1210
    - = $\$1000*1.1*1.1 = \$1000*(1+.1)^2$
  - general formula for compound interest....

$$N(t)=N_0(1+I)^t$$

(t=# of times compounded)

- Example #2: Population Growth (continuous exponential)
  - # of babies born proportional to # of potential parents
  - k is constant of proportionality (e.g. fraction having offspring each year)
  - $dN(t)/dt = k*N(t)$ (differential equation)
  - solve by integrating... $\int(1/N)\,dN = \int k\,dt$
  - so $\ln(N) = kt$; undo natural log with exponential
  - so general formula for exponential growth is ....

$$N(t)=N_0e^{kt}$$

t=time (continuously varies)

Doubling Time

- $N(t)=N_0\, e^{kt}$
  - $2N_0/N_0 = 2 = e^{kt_D}$ [undo exponent with log]
  - $t_D = \ln(2)/k = 100*\ln(2)/100*k$
  - $\ln(2)=0.693$
  - $t_D \approx 70/k$ [where k is in percent per time period]
- $N(t)=N_0*(1+I)^t$
  - $2N_0/N_0 = 2 = (1+I)^{t_D}$ [undo exponent with log]
  - $\ln(2) = t_D \ln(1+I)$
  - $t_D = \ln(2)/\ln(1+I) \approx \ln(2)/I = 100*\ln(2)/100*I$
  - $t_D \approx 70/I$ [where I is in percent per time period]

Finite Resources

- The vast majority of our energy is released by the burning of "fossil fuels"
- We *process* (with a significant energy cost; around 25% ?) these fuels to make them more useful, but they are naturally produced
- Nature takes 100's of millions of years to renew fossil fuels; so they are *non-renewable* on human timescales
- They are therefore a *"finite"* resource

Lifetime of Finite Resource

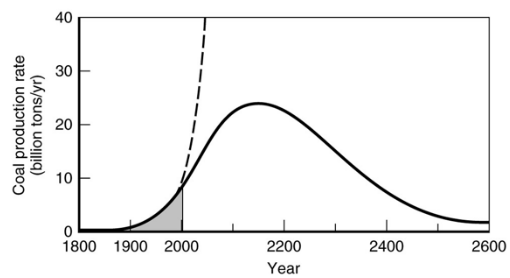
- Lifetime =
  (Amount Available) / (Consumption Rate)
  - e.g. 16 gallon tank / 2 gallons per hour --> 8 hours
- But consumption rate is not constant!
- What does this do to the lifetime?
- <u>growth</u> in consumption -> <u>decrease</u> in lifetime
- This simple fact is perhaps the most overlooked and misunderstood aspect in public and social policy regarding energy
- We have even less time than you realize!

How Do We Estimate Lifetime?

1. assume resource is infinite
   - discoveries must keep pace with consumption
2. deplete at constant amount (current use rate)
   - must decrease per capita use at same rate as population increases
   - production must maintain current pace
3. exponential growth until resource expires
   - production rate must also increase exponentially
4. Hubbert model
   - early exponential rise
   - production slows & peaks when 1/2 resource is consumed
   - steady decline in production rate
   - symmetric, bell-shaped curve

Example 1 (infinite resource) - What's wrong with this picture?

Thought experiment: assume Earth's interior is 100% coal (or oil) . How long will it sustain exponential growth?



Growth *Rate* is What Matters !

- Assume entire Earth is made of petroleum
- $N_T = 4/3\ \pi\ R^3 = 1\ E\ 21\ m^3$
- $N_0 = 1E12$ bbl = $1.6\ E\ 11\ m^3$
- or even assume $N_0 = 1\ m^3$
- how long would it take to drain the Earth?

k	N_0= 1E12	N_0=1
1%	1804 years	4383 years
2%	937 years	2226 years
7%	286 years	654 years
10%	203 years	461 years
25%	85 years	188 years

How Do We Estimate Lifetime?

1. assume resource is infinite
   - discoveries must keep pace with consumption
2. deplete at constant amount (current use rate)
   - must <u>*decrease*</u> per capita use at same rate as population <u>*increases*</u> (increased efficiency and/or lifestyle changes)
   - production must *maintain* current pace
3. exponential growth until resource expires
   - production rate must also increase exponentially
4. Hubbert model
   - early exponential rise
   - production slows & peaks when 1/2 resource is consumed
   - steady decline in production rate
   - symmetric, bell-shaped curve

Lifetime of Current "Reserves" (assuming constant consumption)

Table 1.1 WORLD AND UNITED STATES PROVEN RESERVES: 2008

Resource	World	United States	Lifetime*
Oil	1342×10^9 bbl 7.7×10^{18} Btu	29.4×10^9 bbl 0.13×10^{18} Btu	10 years
Natural gas	6254×10^{12} cf 6.1×10^{18} Btu	237×10^{12} cf 0.24×10^{18} Btu	12 years
Coal	0.93×10^{12} tons 23×10^{18} Btu	0.26×10^{12} tons 6.4×10^{18} Btu	230 years
Oil sands	525×10^9 bbl 2.9×10^{18} Btu	32×10^9 bbl 0.17×10^{18} Btu	12 years

*Ratio of U.S. reserves to 2008 U.S. production rate

How Do We Estimate Lifetime?

1. assume resource is infinite
   - discoveries must keep pace with consumption
2. deplete at constant amount (current use rate)
   - must decrease per capita use at same rate as population increases
   - production must maintain current pace
3. exponential growth until resource expires
   - production *rate* must also *increase* exponentially
4. Hubbert model
   - early exponential rise
   - production slows & peaks when 1/2 resource is consumed
   - steady decline in production rate
   - symmetric, bell-shaped curve

Exponential Expiration Time

- $T_{exp} = (1/k) \ln \{kN_T/N_0 + 1\}$
  - comes from integrating exponential growth:
  - $dN(t)/dt = k*N(t)$
  - $N(t)=N_0 e^{kt}$
  - $N_T = \int^{T_{exp}} N_0 e^{kt} dt$
- Must be able to extract resource as fast as it is needed. But...

  "oil doesn't come from a hole in the ground, it comes from rocks" (Kenneth Deffeyes)

Example 2: constant

T_{exp} = amount left / current rate

per capita use must decrease exponentially if population grows exponentially

Example 3: exponential

$T_{exp} = (1/k) \ln \{kN_T/N_0 + 1\}$

but nature and economics won't allow it!

Example 4: Hubbert

$T_{exp} = \infty$!!

but, that's not the issue!